Виж Crux Mathematicorum 3/1997, стр. 179.

2130. [1996:123] *Proposed by D.J. Smeenk, Zaltbommel, the Netherlands.*

A and *B* are fixed points, and *l* is a fixed line passing through *A*. *C* is a variable point on *l*, staying on one side of *A*. The incircle of ΔABC touches *BC* at *D* and *AC* at *E*. Show that line *DE* passes through a fixed point.

Solution by Mitko Kunchev, Baba Tonka School of Mathematics, Rousse, Bulgaria.

We choose the point *P* on *l* with $AP = AB$. Let *C* be an arbitrary point of *l*, different from *P* but on the same side of *A*. The incircle of ΔABC touches the sides *BC*, *AC*, *AB* in the points *D*, *E*, *F* respectively. Let *ED* meet *PB* in the point *Q*. According to Menelaus' Theorem applied to ΔCBP and the collinear points *E*, *D*, *Q* we get

$$\frac{PE}{EC}\cdot\frac{CD}{DB}\cdot\frac{BQ}{QP}=1. \qquad (1)$$

We have $EC = CD$ (because they are tangents from *C*). Similarly, $AF = AE$, so that $FB = EP$ (since $AB = AP$). But also, $FB = DB$, so that $DB = PE$. Setting $EC = CD$ and $DB = PE$ in (1), we conclude that $BQ = QP$; therefore *Q* is the mid-point of *BP*. Hence the *DE* passes through the fixed point *Q*.